

# Maths vs (Meta)Modelling : Are we reinventing the Wheel?

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**Abstract.** In the past, specification of languages and data structures has traditionally been formally achieved using mathematical notations. This is very precise and unambiguous, however it does not map easily to modern programming languages and many engineers are put off by mathematical notation. Recent developments in graphical specification of structures, drawing from Object-Oriented programming languages, has led to the development of Class Diagrams as a well-used means to define data structures. We show in this paper that there are strong parallels between the two techniques, but that also there are some surprising differences!

## 1 Introduction

Computer Science originally evolved as a branch of mathematics. It could well be argued that maybe it should always have stayed a branch of mathematics because then our programs would be 'proved' to work before being executed. However, the set of programs for which we can 'prove' things is much smaller than the set of programs that we actually want to write, and more importantly, smaller than the set we want to use.

If mathematics were the language of programming, either there would be a much larger number of mathematicians in the world or computers and software would simply not have permeated our culture as much as they have. The traditional mathematical approach to writing algorithms is declarative, whereas the typical programming approach is imperative. This, in our opinion, is one of the most significant differences between mathematics and programming. A consequence of this

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"To be honest, I would have never invented the wheel if not for Ugg's ground breaking theoretical work with the circle!"

difference is an increase in the semantic complexity in traditional programming languages compared to the semantic simplicity of declarative languages.

The complexity of the programs we write naturally leads us to the need for modelling. Modelling is an old discipline, possibly as old as engineering in general. The actual age of modelling is dependant on what one takes as the definition of modelling. Physical sculpture as modelling is pre-historic, mankind has been fashioning models of things in the real world for as far back in his history as he can look.

Modelling as an engineering discipline is also a significantly old discipline; any engineering project involves construction of a model, for small projects this may only be a 'mental model' but for any project of significant size or complexity, especially if it involves multiple engineers, a more concrete, real world, model is created in order to aid communication and exploration of the problems and specification of the final product. Models strive to efficiently communicate the important abstract properties of the problem being modelled.

Mathematics has traditionally been used as a tool for constructing such models. Mathematical models of the stock market, of the weather system, of the forces involved in sending a rocket to the moon, they are all essential to mankind's understanding and ability to interact with, build, or predict things about, the environment in which he lives. Programming, on the other hand, is simply a tool. It is, at the simplest level, a set of instructions for a machine to execute in order that the machine performs some useful task (though in some cases the actual usefulness is dubious). The complexity and the variety of the tasks to which we want to put our machines are increasing astonishingly quickly, thus it becomes more and more essential for us to be able to understand, predict and communicate about the programs that are written; hence the use of models.

Models, in the sense of the Unified Modelling Language (UML) [1, 2], have evolved as the non-mathematician, software engineer's tool for facilitating communication and analysis of the complex programs that they build. A huge part of the modelling languages developed for this purpose focus on the structural elements of the program rather than the behaviour, which after all is the main purpose of the program. This split between structure and behaviour, is in itself a very interesting topic for discussion, however, although we may touch on it in this paper it is not the primary focus.

Techniques and languages for modelling software have changed over the years to reflect the programming languages in common use. Early modelling approaches of flow charts, structure diagrams, data flow modelling, have been replaced with the current favoured approach of Object-Oriented modelling. Although the UML consists of multiple different modelling languages with different modelling features for example: state based modelling, component based modelling, and activity flow based modelling. The core and most widely accepted and used part of the UML is the humble Class Diagram. This diagram type is fundamentally based on the notions of object-orientation: composition, abstraction, inheritance, modularity, polymorphism and encapsulation.

The UML Class Diagram language arose towards the end of the 1990's, as a result of the coming together of (initially) three different languages that had been separately developed for a very similar purpose. Booch's development focussed approach (Booch method) closely related to OO programming; Rumbaugh's Object Modelling

Technique (OMT) coming from the Relational Database world; and Jacobson's Use Case based approach, OOSE.

The primary case study carried out and published as part of the definition of the "new" Unified Modelling Language was the definition of itself! Thus right from the very start, the UML (predominantly class diagrams) has been used as a language to model languages. Such a model of a language has come to be known as a metamodel (a model of a model).

The use of UML as a means to model languages has been part of the fuel for the recent advances in the OO modelling community, and in particular Model Driven Development (MDD [3-5]) research, which has inspired a new interest in language specification. This new interest comes under the title of Domain Specific Languages (DSL [6-10]). As a result, numerous languages are being defined, and in particular, numerous metamodels for those languages.

Prior to UML alternative (traditional) modelling techniques were employed to significant effect in order to define languages. Set theory, logic, and other branches of mathematics were used to give precise and formal definitions of languages including their semantics. These languages were predominantly text based and Backus-Naur-Form (BNF) is used to define the language syntax. It is useful at this point to note a significant difference, to a language specification reader, between a BNF grammar and a UML class diagram. In BNF, the syntax is presented in an entirely text based format and although complete and theoretically fit for purpose, it presents a possible conceptual barrier to the ease of understanding for a typical human reader. Further, BNF is overly specific regarding the nature of the syntax whereas the graphical based format of UML primarily introduces the abstract concepts in an easily accessible and pictorial manner. Recent works such as [11, 12] explore the relationships between BNF based definitions of syntax and metamodels.

Mathematical modelling of algorithms has evolved to a very high degree. Denotational semantics [13, 14] allows a detailed analysis of algorithms to be made and conclusions to be drawn regarding their behaviour and efficiency. However, the very mathematical nature of Denotational Semantics makes it highly inaccessible to traditional programmers and therefore its practical uptake has been limited. Alternatively, many practical logics have been developed for specific problem domains and theorem proving tools designed [15-17] to help verify and validate software systems. Again however problems associated with the complexity of such systems has limited their practical employment and in some cases has actively hindered the verification process due to errors introduced by human operators within the program validation stage [18].

The primary aim of the paper is to investigate the use of the graphical Object-Oriented approach of metamodelling in contrast to traditional approaches for the specification of languages. It seeks to compare the practical issues related to clear and precise modelling offered by the mathematical techniques with the human accessibility and, by implication, practical utility, offered by the graphical approach. To achieve this aim, three contrasting examples are discussed in the following sections. Section 2 introduces the simplistic notion of a Directed Graph. Subsequently section 3 enhances the discussion to Petri-Nets and Section 4 addresses a significantly different example in the form of the Lambda Calculus. The paper then draws conclusions based on the relative merits of the proposals in Section 5.

## 2 Modelling Directed Graphs

One of the most common structures used in both mathematics and computing is that of a directed graph. A graph  $G=(N,E)$  is a pair of sets;  $N$  is a finite set of nodes or vertices and  $E$  is a set of pairs of elements of  $N$ . In depiction of graphs, nodes are points in some space and edges form connections from one node to another.

A very simple mathematical model of such a graph is shown in Table 1. This specifies the concept of a directed graph as a *pair* of sets. One set being a set of nodes, which for simplicity are represented by integers. The other set, representing the edges in the graph, is a set of pairs (tuples); each of the pairs containing two integers representing the nodes which are connected by the edge.

$G = (N, E)$ $N \subseteq \{ n \mid n \in \mathbb{Z} \}$ // where $N$ is finite $E \subseteq \{ (n1,n2) \mid n1,n2 \in N \}$
--

Table 1 Mathematical Model of a Directed Graph

To model the same kind of structure using OO modelling, Class Diagrams, we would typically define classes DirectedGraph, Node and Edge, and then define associations between the classes that indicate the relationships between nodes and edges and the graph as a whole. This is illustrated in the simplest form by Figure 1.

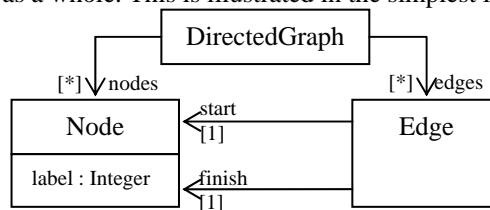


Figure 1 Typical OO Model of a Directed Graph

Interestingly, even with this simple graph structure, there are significant differences between the mathematical and OO forms of model. If we interpret each class in the typical OO programming context, then there is an implicit property of each class/object (equitable to the memory location of the object representation) that defines the objects identity. In the case of the Node class, a 'label' property has been given, but there is nothing in the model that specifies that this property defines the identity of Node objects.

In the mathematics, the identity of the nodes ( $N$ ) are made explicit by defining a node as an integer; however, neither graph ( $G$ ) nor edge ( $E$ ) have an explicit identity other than the implicit identity of the tuple and set on which they are defined.

With respect to the OO model the question arises as to whether two edge objects that refer to the same two nodes are a single edge or two separate edges. In the mathematics, the definitions of identity for tuples clearly indicates that, not only are two pairs of the same two nodes, the same edge, but that a graph cannot have two edges between the same two nodes (something can only appear once in a set.)

Another difference is that the mathematical model specifies that the two ends of an edge are members of the set of nodes. The OO model does not make this restriction;

the edges in this model may be node objects that do not appear in the set of nodes for the graph.

So is our mathematical model wrong, or is it the OO model that is incorrect? Neither, they just happen to model two different structures. A mathematical model of the same structure as defined by the OO model of Figure 1 is shown in Table 2.

$\text{Graph} = (\text{id}, \text{nodes}, \text{edges})$ $\text{id} \in \mathbb{Z}$ $\text{nodes} \subseteq \{ (\text{id}, \text{label}) \mid \text{id}, \text{label} \in \mathbb{Z} \}$ $\text{edges} \subseteq \{ (\text{id}, \text{n1}, \text{n2}) \mid \text{id} \in \mathbb{Z}, \text{n1} \in \text{Node}, \text{n2} \in \text{Node} \}$
---

Table 2 Mathematical version of OO Model of a Directed Graph

In this structure we have explicit modelled the ‘memory location’ identity as an Integer, and we define edges to reference two elements of the set of all Node objects as opposed to two elements from the set of nodes in the graph.

An OO model that provides a slightly better match to the original mathematical definition from Table 1 is illustrated in Figure 2.

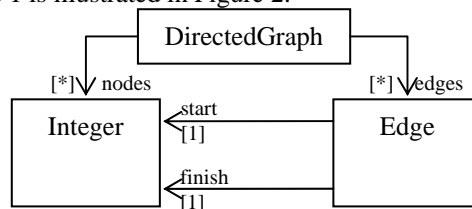


Figure 2 Another OO Model of a Directed Graph

This second OO model directly models the nodes of a graph as a set of integers. However, using this simple class diagram language, there is no way to provide a completely equivalent model:

1. We have no means to define the identity of classes/objects. In the case of an Integer, one has to assume that its value is its identity, but for the Graph and Edge classes, there is the implicit notion of ‘memory location’ identity which we have no means to override. Ideally we would define the identity of an edge as being equivalent to the set of its ends.
2. We also have no way to define that the ends of an edge must be a subset of the nodes in the graph.

To enable the precision of specification easily achieved with the mathematical approach, we must add a means for defining/overriding the identity of objects and a way to add constraints.

Both of these things have been addressed (to an extent) by the designers of the UML. Additional constraints can be added to a model using the OCL, and there is a basic mechanism for defining that certain properties of a class define its identity<sup>1</sup>. Using this extended language of class diagrams we can now give an equivalent

<sup>1</sup> We extend the official UML facility slightly, allowing us to mark multiple properties as jointly defining an object’s identity and allowing those properties to be association ends as well as attributes.

structural specification of our original mathematical definition of a graph, shown in Figure 3.

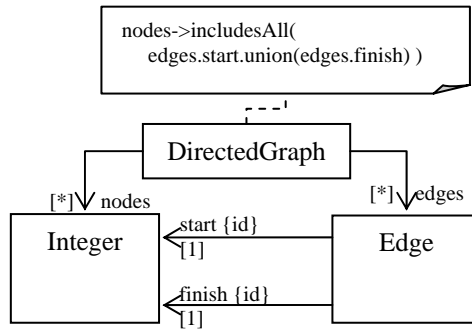


Figure 3 OO version of Mathematical model of a Directed Graph

This OO model of a directed graph is a much more precise specification. However, even though it does match the mathematical definition, it seems somewhat clumsy with the need for the additional constraint. Also the use of the Integer class directly for modelling nodes does not seem quite like the OO approach to modelling.

If we make use of bi-directional associations and extend the use the UML 2.0 notion of *subsetting*<sup>2</sup> we can construct a new model as illustrated in Figure 4. In this model:

- Node objects are identified by their label – the *label* property is marked as an identifying property.
- Edges are identified by the nodes they connect – the *start* and *finish* properties are marked as identifying
- The ends of an edge are constrained to be nodes in the same graph – the *start* and *finish* properties are constrained to be subsets of the *nodes* in the *graph*.

In addition the model provides bi-directional navigation between Node objects and the edges that connect them, which although non-essential is likely to be very useful.

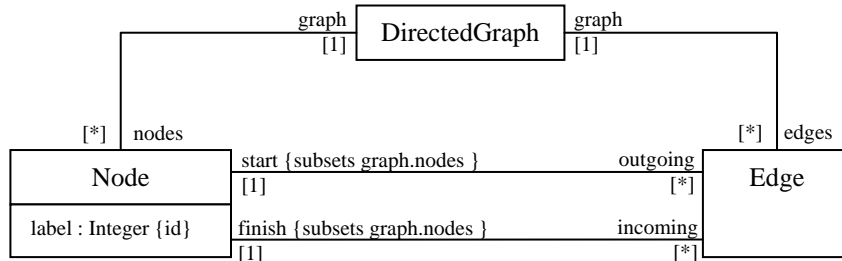


Figure 4 Better OO model of Directed Graph

This final OO model of directed graphs is, in our opinion, by far the most effective model. The initial OO model, although simple and intuitive, was not precisely correct. By looking at the traditional mathematical model it becomes apparent that the notion

<sup>2</sup> Officially, association ends should only subset other associations ends with the same source object.

if identity is important, as is a mechanism for constraining the ends of edge objects to be part of the same graph as the edge itself.

There are mechanisms designed into the UML language of class diagrams that nearly enable us to model as precisely as the traditional maths, however these notions are not quite sufficient, and more importantly are seen as ‘additions’ to an OO model rather than primary things to consider.

This example has looked solely at models of structure (a graph has no behaviour). In the next two sections we look at modelling more complex structures (languages) and at specifying some aspects of the semantic behaviour implied by those structures.

### 3 Petri-Nets

A more interesting example than simple graphs is that of Petri-nets [19]. A Petri-net is directed, weighted, bipartite graph, together with an initial state called the *initial marking*,  $M_0$ . Petri-net graphs consist of two types of nodes, called *places* and *transitions*, whereas edges are either from places to transitions or from transitions to places. If there is an arc from a place  $p$  to a transition  $t$ , we say  $p$  is an *input* place of  $t$ , and  $t$  is an *output* transition of  $p$ . Places are depicted as circles and transitions as rectangles. Arcs are labelled with positive integers, called *weight*. A marking of a Petri-net assigns a non-negative number, known as the number of *tokens*, to each place. A marking  $M$  is in effect an integer valued vector of dimension  $m$ , where  $m$  is the number of places. Hence, each coordinate of  $M$  denotes the number of the number of tokens in the corresponding place. Table 3 presents a formal definition of a Petri-net, taken from [19].

$PN = (P, T, F, W, M_0)$ $P = \{p_1, p_2, \dots, p_m\}$ is finite set of Places $T = \{t_1, t_2, \dots, t_n\}$ is a finite set of transitions $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs $W: F \rightarrow \{1, 2, 3, \dots\}$ is a weighting function $M: P \rightarrow \{0, 1, 2, \dots\}$ is a marking. $M_0$ is the initial marking.
---

Table 4 Mathematical definition of a Petrinet

An OO model would be more likely to define Petri-nets as illustrated in Figure 5. A Petri-net being a containing class for Place, Transition and Arc objects. There being two kinds of Arc, Place->Transition Arcs and Transition->Place Arcs. The additional definition of Markings is as shown in Figure 6.

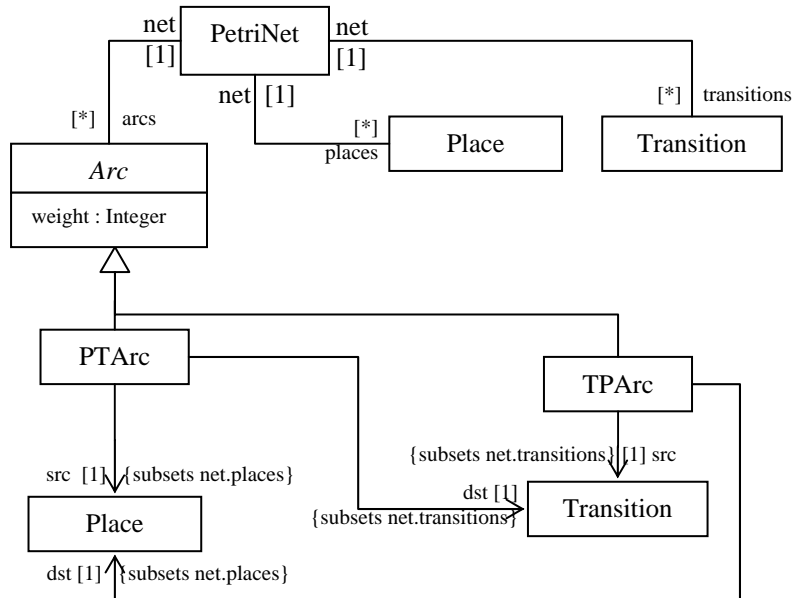


Figure 5 OO (Meta) Model of Petri-Nets

As with the directed graph definition, it is necessary to augment the class diagram with additional constraints, which in the mathematical model are unnecessary. In this case the constraints ensure that the ends of the arcs are members of the sets of places and transitions in the petri-net.

The other major difference between these two specifications is the explicit definition of types for Arcs in the OO model, which in the maths specification are defined jointly as the union of tuples (place,transition) and (transition,place).

### 3.1 Semantics

With the definition of a language, in this case Petri-nets, we can go a step further than we did with the graph model. The runtime semantics of Petri nets are defined using the traditional mathematical specification approach. In the OO modelling world this is less often defined; however, it is perfectly feasible to do so, using the standard UML/OCL language facilities.

Semantics of a Petri net can be interpreted as a labelled transition system, in which each *state* of the Petri is a marking of the Petri net. Change of one state of a Petri net to another state is governed by the *firing* rules:

- 1) A transition *t* is called *enabled* if for each of its input places *p* has at least  $w(p,t)$  tokens, where  $w(p,t)$  is the weight of the arc from *p* to *t*.
- 2) An enabled transition *t* may fire, in which case  $w(p, t)$  tokens are removed from each input place *p* of the transition *t* and  $w(t, p')$  tokens are added to the each output place *p'* of *t*.

Firing of an enabled transition *t* under a marking *M* resulting in new marking *M'* is denoted by  $M \xrightarrow{t} M'$ . A *Reachable* marking (state) of a Petri net is a marking  $M_k$



such that there are marking  $M_1, M_2, \dots, M_{K+1}$  and transition  $t_1, t_2, \dots, t_k$  satisfying the following

$$M_0 [t_1 > M_1 [t_2 > M_2 [t_3 > \dots [t_{K+1} > M_{K+1} [t_K > M_K$$

In this case the sequence of transitions  $t_1 t_2 \dots t_k$  is called a *run*.

To add this semantics to the OO model of Petri-nets we can define operations on the classes that provide the firing behaviour, the body of the operations can be given using OCL expressions<sup>3</sup>. To facilitate more concise OCL expressions, we would also adapt the OO model making more use of bi-directional associations. Figure 6 shows an evolved specification of the model from Figure 5, including the specification of Markings and use of bi-directional associations. (The constraints have been left out for clarity.)

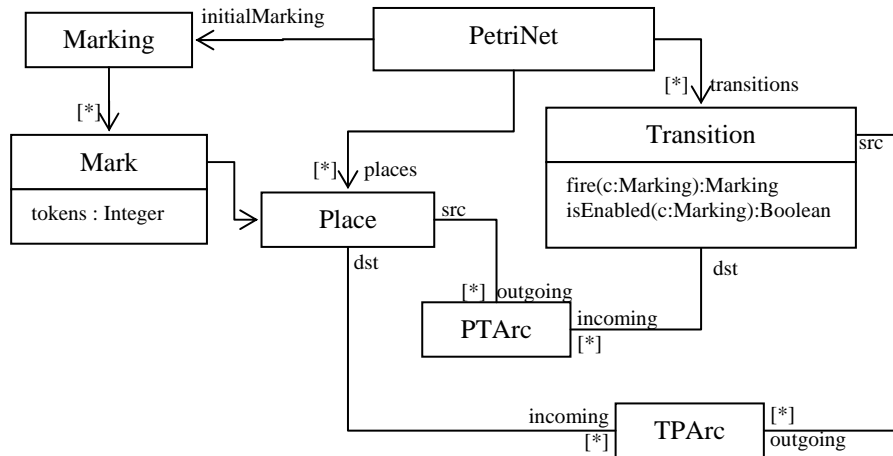


Figure 6 OO (Meta) Model of PetriNets

Given this specification of the model of a Petri-net, we can define the behaviour of the operations as shown in Table 5.

```

context Transition::isEnabled(current:Marking) : Boolean
body: incoming->forAll( arc |
  let mark = current.mark->any(m|m.place=arc.src) in
  mark.tokens >= arc.weight
)

context Transition::fire(current:Marking) : Marking
body: let
  unaffected = current.mark->reject( m |
    incoming.src->union(outgoing.dst)->includes(m.place) ),
  lost = incoming.src->collect( arc |
    let mark = current.mark->any(m|m.place=arc.src) in
    Mark { place = arc.src,
           tokens = mark.tokens-arc.weight } ),
  gained = outgoing.dst->collect( arc |
    let mark = current.mark->any(m|m.place=arc.src) in
  
```

<sup>3</sup> We find it necessary to use an extension of OCL that allows us to create instances of user model objects. Creation of such objects is similar to the creation of tuples in standard OCL.

```

Mark { place = arc.dst,
      tokens = mark.tokens+arc.weight } ),
in
Marking {
  mark = unaffected.union(lost).union(gained)
}

```

Table 5 Definitions for Petri-net behaviour

Using these definitions we could define further operations that would simulate execution of the Petri-net, or search the reachable Markings. One could even go so far as to build various model-checking operations.

It can be seen from these specifications that a graphical OO definition of the language can be as precise as the more traditional definition. It is also possible, using the OO approach, to define operations that aid the semantic interpretation of the language.

One distinct advantage of the OO definition, over the traditional, is that the MDD and code generation techniques ([20, 21]) enable this definition of the language to be used to automatically produce an executable version of the model that can be used as a first cut evaluator for the language.

Another, more subjective, observation is that the graphical specification of the concepts of Place, Transition and Arcs conveys more information to the reader about the structure of expressions in the language than does the text based more traditional specification.

## 4 Untyped Lambda Calculus

The third example we will look at is one of the fundamental languages of computer science - lambda calculus. The definition of this language (taken from [22]) is given as a grammar shown in Table 6. The definition of this language differs from the previous two examples, in that the language definition is given using its syntax, in BNF, rather than being a set theory based definition.

```

expr = ID
      | 'λ' ID '.' expr
      | expr expr
      ;

```

Table 6 Grammar for Lambda Calculus

Further definition of the language is then given using text definitions and illustrated by examples. The semantic of the language are given by defining the notions of substitution ( $\alpha$ -conversion) and  $\beta$ -reduction. These definitions are shown in Table 7, taken directly from [22].

The **substitution** of  $f$  for the free occurrences of  $x$  in  $e$ , written  $e[f/x]$  is defined thus.

- $x[f/x] \equiv_{df} f$  and for a variable  $y \neq x$ ,  $y[f/x] \equiv_{df} y$
- For applications, we substitute the two parts:  
 $(e1 e2)[f/x] \equiv_{df} (e1[f/x] e2[f/x])$
- If  $e \equiv \lambda x.g$  then  $e[f/x] \equiv_{df} e$ . If  $y$  is a variable distinct from  $x$ , and  $e \equiv \lambda y.g$  then  
- if  $y$  does not appear free in  $f$ ,  $e[f/x] \equiv_{df} \lambda y.g[f/x]$ .

<p>- if <math>y</math> does appear free in <math>f</math>, <math>e[f/x] \equiv_{df} \lambda z.(g[z/y][f/x])</math></p> <ul style="list-style-type: none"> <li>• In general, it is easy to see that if <math>x</math> is not free in <math>e</math> then <math>e[f/x]</math> is <math>e</math>.</li> </ul> <p>The rule of <b><math>\beta</math>-reduction</b> states that, for all <math>x</math>, <math>e</math> and <math>f</math>, we can reduce a function application by substituting the argument for the bound variable</p> <ul style="list-style-type: none"> <li>• <math>(\lambda x.e) f \rightarrow_{\beta} e[f/x]</math></li> </ul> <p>And if <math>e \rightarrow_{\beta} e'</math> then</p> <ul style="list-style-type: none"> <li>• <math>(f e) \rightarrow_{\beta} (f e')</math></li> <li>• <math>(e g) \rightarrow_{\beta} (e' g)</math></li> <li>• <math>\lambda y.e \rightarrow_{\beta} \lambda y.e'</math></li> </ul>
--

Table 7 Definitions of substitution and reduction

Using OO modelling techniques we can provide equivalent definitions, but using a metamodel of the lambda calculus concepts, and OCL to define the substitution and reduction functions. A mapping can be given from a concrete syntax to the metamodel, but the details of this are not in the scope of this paper. A metamodel for the Lambda calculus is given in Figure 7. It shows an abstract *Expr* type which is realised by the three kinds of expression that can be formed, a function *Application*, a *Variable*, and a function *Abstraction*.

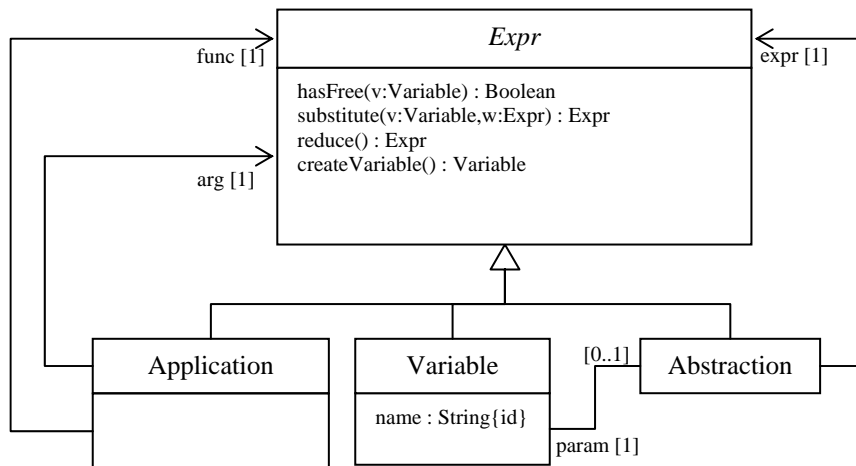


Figure 7 OO (Meta) Model of Lambda Calculus

Based on this metamodel, the notion of substitution can be defined as an operation that returns a new Expr. The behaviour of such an operation needs to be defined on each kind of expression and these definitions are given in Table 8.

<pre> context Variable::hasFree(v:Variable) : Boolean body: self==v  context Application::hasFree(v:Variable) : Boolean body: func.hasFree(v) and arg.hasFree(v)  context Abstraction::hasFree(v:Variable) : Boolean body: param &lt;&gt; v and expr.isFree(v) </pre>
---

```

context Variable::substitute(v:Variable, w:Expr) : Expr
body: if self==v then w else self endif

context Application::substitute(v:Variable, w:Expr) : Expr
body: Application { func = func.substitute(v,w),
                    arg = arg.substitute(v,w) }

context Abstraction::substitute(v:Variable, w:Expr) : Expr
body: if w.hasFree(param) then
    let z = createVariable() in
    Abstraction { param = z,
                 expr = expr.substitute(param,z).substitute(v,w) }
else
    Abstraction { param = param,
                 expr = expr.substitute(v,w) }
endif

```

Table 8 OCL Definitions for substitution ( $\alpha$ -conversion)

Reduction is the expansion of a function application, substituting the argument for the function parameter. This also can be defined by operations on the Expr subclasses, as shown in Table 9.

```

context Variable::reduce() : Expr
body: self

context Abstraction::reduce() : Expr
body: self

context Application::reduce() : Expr
body: if func.oclIsKindOf(Abstraction) then
    func.substitute(func.oclAsType(Abstraction).param, arg)
else
    self
endif

```

Table 9 OCL Definition for  $\beta$ -reduction

Further concepts such as equivalence, normalization or  $\eta$ -reduction can be defined as additional operations that make use of the reduction and substitution functions. It is interesting to note that the addition of a transitive closure operation within OCL would ease the definition of some of these additional operations.

It is of course a very subjective issue as to whether the traditional BNF based specification of this language is better or worse than the OO version. Your preference as a reader of the definition probably depends largely on your background and previous experience of specifications. However, as is the case with the Petri-net example, this definition can be used to generate an executable model of the language. It is also interesting to see the difference between basing the language definition on the syntax or the concepts. The traditional approach defines the syntax of the language and uses this on which to base the definition of the semantic functions. In contrast the metamodel defines only the concepts of the language (potentially enabling multiple syntaxes), but still provides precise definition of the semantic substitute and reduction functions.

## 5 Conclusion

The paper has employed Object-Oriented graphical specification techniques to model three separate well-known languages or data structures. These examples are

initially specified using traditional mathematical techniques and it has been shown that these may equally well be expressed using the O-O graphical methods. Further, the paper has sought to demonstrate the increased ease of comprehension of the O-O graphical techniques by contrasting the two alternative specification techniques.

The major observations may be summarised as follows:-

- It is surprising that OO modelling seems to have forgotten the importance of identity. Notions of identity, naturally assumed in mathematical specifications are not the same as the default notions in OO models. Relational modelling, itself an ancestor of UML, in contrast, makes this importance clear via primary keys.
- An important advantage of the OO graphical approach may be deduced by the assumption that mental pictures will be created by a reader when digesting a specification. When reading mathematical or text based specifications, the mental picture is constructed by the reader. In contrast, by explicitly giving the picture as part of the specification (class diagram) this helps the specification writer ensure that his own mental picture is better communicated to the reader. This significantly improves the ability to mentally communicate and/or interpret the abstract concepts involved.
- By modelling languages using class diagrams we gain the added advantage of being able to automatically generate tool support for the newly created language. The Model Driven Development (MDD) and code generation techniques developed for aiding rapid development of general software systems can be readily employed as part of the Domain Specific Languages (DSL) or grammarware engineering [23] discipline supporting the rapid development of tools to support new or new versions of a language. Such support is not generally available if given a specification in a traditional mathematical formalism, other than perhaps that given by compiler compilers.
- OO Modelling gives a more complex set of basic concepts for producing models, whereas mathematics uses a much simpler set. For example, notions of extensibility in OO techniques are not generally primitive concepts in traditional specification techniques. Due to the simplicity of the primitives used within the mathematical models, the expressions tend to be more precise and unambiguous.
- Ironically, the ease of comprehension possessed by OO graphical techniques means that a human reader is likely to infer fewer ambiguities in this presentational style than would be the case for the mathematical techniques, even though the mathematical techniques will actually contain fewer ambiguities.

Metamodelling is a practical engineering approach to modelling a language whose primary goal is to aid the designer in producing a working solution to a problem. In contrast, the mathematical approach is primarily driven by the need for precision and accuracy rather than practical utility. Although metamodelling can be as precise as a mathematical approach, some of the underlying concepts do not encourage this precision.

So, is metamodelling reinventing the wheel? Yes, but the wheel is a different colour! Specifically, many of the same concepts are available but their utility is

improved by the improved accessibility of the concepts concerned. I.e. this colour of wheel is easier on the eye!

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